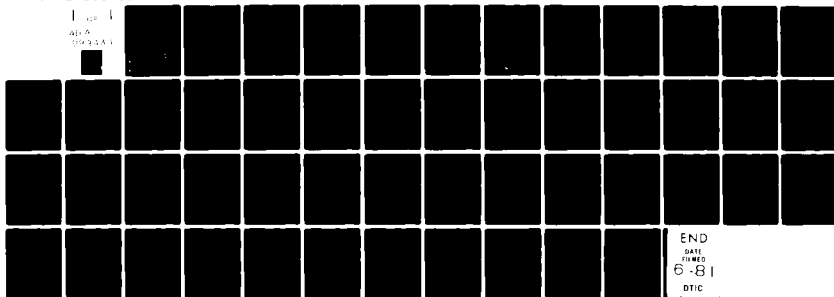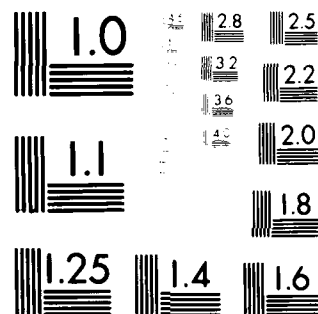AD-A099 439   AEROSPACE CORP EL SEGUNDO CA ELECTRONICS RESEARCH LAB   F/G 20/12
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Statistical Analysis of Accelerated
Temperature Aging of Semiconductor Devices

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15 May 1981

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A statistical error analysis of accelerated temperature-stress testing of semiconductor devices is presented. Our primary interest is to estimate the expected lifetime of semiconductor components and its associated statistical error under normal operating conditions on the basis of information derived from an accelerated temperature-aging program. On the assumption that n devices, all of which failed at a constant temperature, follow a lognormal probability density function, the probability that an n + 1 device, randomly

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20. ABSTRACT (Continued)

chosen from the same distribution, will have a lifetime shorter than a preselected value is

$$P\left[\ln t < \bar{\mu} - t(\delta)_{n-1}\left(1 + \frac{1}{n}\right)^{1/2}\hat{\sigma}\right] = \delta$$

where $\bar{\mu}$ and $\hat{\sigma}$ are the estimated median log lifetime and estimated logarithmic standard deviation of the failed sample distribution, and $t(\delta)_{n-1}$ is the Student-t distribution with $n - 1$ degrees of freedom associated with the confidence coefficient δ. On the further assumption that the median log life-time follows an Arrhenius dependence but that the logarithmic variance is temperature-insensitive, the aging to failure of n devices at several elevated temperatures is used to infer the probability that a device operated at some lower temperature will fail before a preselected time. The probability is

$$P\left[\ln t < \bar{\mu}_N - t(\delta)_{n-2}\hat{\sigma}\left(1 + \frac{1}{\eta n}\right)^{1/2}\right] = \delta$$

where $\hat{\sigma}$ is once again our best estimate of the logarithmic standard deviation, and $\bar{\mu}_N$ is the estimated median log lifetime extrapolated to the desired operating temperature employing the Arrhenius relation. The statistical efficiency of the device distribution during accelerated aging is represented by

$$\eta = (DL^2 + 1)^{-1}$$

The experimental uncertainty increases with the difference between the accelerated temperature range and the normal operational temperature and is conveniently expressed by an experimental lever arm, L, which is analogous to the inaccuracy of a rifle decreasing as the ratio of firing range to barrel length. The D-factor accounts for the statistical effectiveness of the device distribution at the accelerated temperatures. Devices that failed at the middle of the accelerated temperature range are statistically inefficient.

The results of this statistical analysis are applied to several typical accelerated temperature-aging programs to demonstrate its application.

CONTENTS

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I. INTRODUCTION

Although accelerated temperature stressing of semiconductor devices is an accepted and widely employed technique for assessing the reliability of semiconductor devices,[1,2] it appears that a statistical error analysis of this procedure is not available. The purpose of this work is to partially remedy this deficiency. The goal of this program is easily stated: A number of devices taken from a distribution are operated at several elevated temperatures to induce failure in all devices within a reasonable time. Assuming general characteristics of the device failure probability density function (pdf) and its temperature-dependence, we estimate the expected cumulative failure function (cff) for devices in normal operation. By estimated cff, we mean our best estimate, based on statistical inference, of the average probability of a random device (taken from the same distribution but operated at a normal temperature) failing as a function of time.

Section II contains a brief review of the general mathematical formalism usually employed in semiconductor reliability discussions. Three failure pdf's ot particular usefulness to this analysis—exponential, normal, and lognormal—are discussed. Our particular interest is in highly complex electronic systems intended for long-life space applications. In this application and for similar ground-based systems, it is not the expected failure probability density, at times comparable to the median lifetime ($\approx 10^7$ to 10^8 hr) of the device that is important, but the cff, at times orders of magnitude less, at times comparable to the desired system useful life ($\approx 10^4$ to 10^5 hr). A simplified cost analysis to justify this assertion is included at the end of Section II. A brief review of accelerated temperature aging is presented, and the assumptions concerning the general characteristics of the failure pdf, which are fundamental to this analysis, are emphasized.

This analysis is carried out in several steps, each more difficult than the preceding. We first consider the case of operating the devices to failure at a single temperature. On the basis of the experimental observed failure times and the assumption of a lognormal pdf, we estimate the lifetime

distribution of the original population from the failed sample distribution. The maximum-likelihood method, combined with the chi-squares and the Student-t distributions, is employed to estimate the median lifetime, logarithmic variance, and their associated distributions, as well as the average cumulative failure function.

This procedure is repeated for a generalized accelerated-temperature-stressing experiment, and these results are first used to analyze a simple two-temperature stress test because the physical understanding is not obscured by the mathematics and also because such a program is statistically very efficient within the constraints of the accelerated temperature range or equivalently within the constraint of limited test time. Various other sample distributions within the elevated temperature range are considered, and it is shown that samples near the center of the accelerated-temperature region contribute (statistically) inefficiently to the accuracy of the extrapolated estimates. The final result of this analysis is an estimate of the average cumulative failure function of a device operated at the actual operating temperature. This estimate is based on the sample failure distribution of a thermally accelerated aging test with no prior knowledge of either the median lifetime or its logarithmic variance assumed.

II. BACKGROUND

The reliability of a given device is conveniently described by its
failure pdf, which, by convention is designated $f(x)$, where x is a random
variable. The differential probability that a randomly chosen device will
fail between x and $x + \Delta x$ is $f(x)\Delta x$. The cff, $F(t)$, is the probability that a
given device will have failed before t and is,

$$F(t) = \int_0^t f(x)dx \qquad (1)$$

The survival probability is one minus the cff, when one assumes, as is done in
the present analysis, that a device can only be in one of two states, i.e.,
fully operational or failed. The failure rate, $\lambda(t)$, is

$$\lambda(t) = \frac{f(t)}{[1 - F(t)]} \qquad (2)$$

A given pdf can be characterized by many different parameters, but only
three of these will be employed in this analysis: mean lifetime, median
lifetime, and the variance of the distribution. The mean lifetime for a given
pdf is designated

$$\bar{t} = \int_0^\infty tf(t)dt \qquad (3)$$

and represents approximately the average life expectancy of a large number of devices taken for the distribution under discussion. The variance is a measure of the "spread" of the lifetimes and is defined as

$$\sigma^2 = \int_0^\infty (t - \bar{t})^2 f(t) dt \qquad (4)$$

In this case, the variance is defined with respect to "linear" time as the random variable. Although this is the usual designation, the choice of the random variable is not unique. With semiconductor devices, it is convenient to specify the variance with respect to the logarithm of operating time as the random variable. The median lifetime is similar to the mean lifetime except that it represents the time at which the cff is equal to 0.5. If a large sample of devices were operated to failure, the median lifetime or log lifetime is approximately equal to the time at which one-half of the devices have failed.

The general properties of three failure pdf which are of particular interest to the present analysis are briefly reviewed.

The exponential failure pdf is

$$f(t) = \lambda \exp(-\lambda t) \qquad (5)$$

where λ is a constant. For any true pdf, we have

$$\int_0^\infty f(t) dt = 1 \qquad (6)$$

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The cff assumes a simple form

$$F(t) = 1 - \exp(-\lambda t) \qquad (7)$$

$$\approx \lambda t \quad \text{for} \quad \lambda t \ll 1 \qquad (8)$$

The failure rate is constant and equal to λ. The mean lifetime and its standard deviation are both equal to the reciprocal of the constant failure rate.

The normal failure pdf is

$$f(t) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{t - \mu}{\sigma}\right)^2\right] \qquad (9)$$

where μ is the mean lifetime and σ^2 the variance. Unlike the exponential distribution, the median lifetime is equal to the mean lifetime. The cff

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{t - \mu}{\sigma}\right)^2 dt\right] \qquad (10)$$

cannot be expressed in a closed form and must be evaluated by using a tabulation of the standard normal distribution

$$F(a) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{(a-\mu)/\sigma} \exp\left(-\frac{1}{2} y^2\right) dy \tag{11}$$

Probability graph paper (e.g., K & E/46 8003) is particularly useful when working with normal distributions. When the cff is plotted on this graph paper, a straight line results. Both the median and the mean lifetime occur at cff = 0.5 (i.e., 50%) and

$$\sigma = \mu - t_{16} \tag{12}$$

where t_{16} is the time corresponding to $F(t) = 0.16$. The normal pdf is symmetric about μ, and approximately 68% of the failures occur within σ of the median lifetime.

Under carefully controlled conditions of elevated temperature and applied bias, the failure pdf of semiconductor devices is found[1,2] to follow a lognormal pdf expressed as

$$f(t) = \frac{1}{t\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{\ln(t/\tau_M)}{\sigma}\right)^2\right] \tag{13}$$

In this expression, τ_M is the median lifetime, and σ is the logarithmic standard deviation. The lognormal distribution is similar to the normal pdf except that $t \to \ln t$. In fact, if we define

$$x = \ln t \tag{14}$$

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we have

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad (15)$$

a normal distribution employing not a linear time scale but a logarithmic time scale. By working in logarithmic time, all the properties of the normal distribution apply to the lognormal pdf. The time interval $[t, t + \Delta t]$ becomes $[x, x + \Delta x] = [\ln t, \ln t + \Delta t/t]$. The cff is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad (16)$$

which, when plotted on probability graph paper using the logarithm of time, results in a straight line. This straight line crosses the 50% probability at $x = \mu$; the median of the log lifetime distribution. The median lifetime is, of course,

$$\tau_M = \exp \mu \qquad (17)$$

In a manner similar to a normal distribution, we have approximately

$$\sigma \approx \mu - x_{16} = \ln\left(\frac{\tau_M}{t_{16}}\right) \qquad (18)$$

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where x_{16} and t_{16} correspond to the value of x and of t where 16% of the cumulative failure occurs. Approximately 68% of the failure occurs between x_{16} and x_{84} (i.e., 2σ, centered around the median lifetime).

Expressions for the real-time (i.e., linear) mean lifetime and variance [i.e., Eqs. (3) and (4)] are available[3] but serve no useful purpose in the present analysis. In fact, we emphasize that knowledge of the failure pdf near the mean lifetime is of little value in analyzing the reliability of complex systems intended for long-life space applications or other similarly complex ground-based systems. Of prime importance is the failure rate at times much less than the average lifetime. In general, with a system consisting of n components, one desires to know the cff near the 100/n percentage level. For example, a system consisting of 10^4 identical components in series would be expected to have a high probability of failure at a time when the average device cff is 10^{-4}.

If we look at a total system from the viewpoint of cost-effectiveness, it is not the failure pdf or cff that one is interested in, but rather the integral of the cff over the service life. Consider a system with a specified useful life of t_s. Let the loss in some monetary units for the systems not working in the time interval $[t, t + \Delta t]$ be $R(t)\Delta t$. The expected loss from failure of the ith component is

$$\Delta L_s = \int_0^{t_s} R(t)F_i(t)dt \qquad (19)$$

where $F_i(t)$ is the cff for the ith component. The loss rate, of course, depends on the system-intended employment and reliability. Assuming a constant "usefulness" throughout the system lifetime, the loss rate is expected to be proportional to the survival probability of the system excluding the ith component. Assuming a high survival probability for the total system, the loss due to the failure of any component is independent of

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the failure of other components. If the rate of loss R(t) can be reasonably
represented as a constant, $\overline{R}$, we estimate

$$\Delta L_s = \overline{R} \int_0^{t_s} F_i(t)dt \qquad (20)$$

or the total estimated loss from all components, assuming $\Sigma F_i(t) \ll 1$ for $t <$
t_s, is the summation of the individual losses and is

$$L_s = \overline{R} \sum \int_0^{t_s} F_i(t)dt \qquad (21)$$

For an exponential pdf, this takes a particularly simple form assuming
$\lambda_i t \lll 1$, and we write

$$L_s = \frac{1}{2} \overline{R} t_s^2 \sum \lambda_i \qquad (22)$$

i.e., one simply sums the various failure rates. For a lognormal
distribution, a much more complex situation results because a single lognormal
cff, let alone the sum of lognormal cff's, cannot be integrated to yield a
closed form solution.

Thus far, we have been speaking as if we knew the failure pdf
accurately. In practice, we do not know the failure pdf exactly; in fact, in
many cases, we do not even know the functional form of the pdf. Only by
sampling from the distribution can we estimate the correctness of an assumed
functional form, and only when this is established are we in a position to
estimate the parameters associated with the failure pdf. Knowing the
functional form of the failure pdf, one estimates the expected cff from the
sample distribution and not the exact cff. In the present analysis, it is
important to distinguish between the parameters that characterize an exact pdf
and the corresponding parameters estimated from sampling.

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III. ACCELERATED TEMPERATURE TESTING

Because semiconductor components have excellent reliability, it is very costly to adequately determine the failure pdf under actual operating conditions. This excessive cost is associated with the large sample of devices that must be operated for long test times to adequately predict the failure pdf with a reasonable confidence level. Even if the cost of such a reliability test is not a constraint, the required total test time is usually a constraint. If one desires to predict the reliability of an electronic system with a desired system life of t_s, one must determine the failure rate at times comparable to t_s, which for space applications can be ten years or longer. If the failure pdf is exponential, one can decrease the total test time by increasing the sample size, since in this special case, it is the total operating time of the device that establishes the accuracy of predicting the failure pdf. When the functional form of the failure pdf is unknown, it is dangerous to assume that one can trade sample size against test duration.

Because of this consideration, one often resorts to an accelerated temperature-stress program to estimate the reliability of semiconductor components. The success of such a program is based on the remarkable experimental observations[1,2] that for a single failure mechanism

1. The failure pdf at a constant temperature and applied electrical stress is lognormal.

2. The logarithmic variance is independent of temperature.

3. The median failure time follows an Arrhenius dependence expressed as

$$\tau_M = \tau_o \, \exp\!\left(\frac{\Delta E}{kT}\right) \tag{23}$$

where τ_o and ΔE, the activation energy, depend on the electrical stress but not on temperature. A typical accelerated test is schematically shown in Fig. 1, in which a small sample of devices are operated to complete failure at
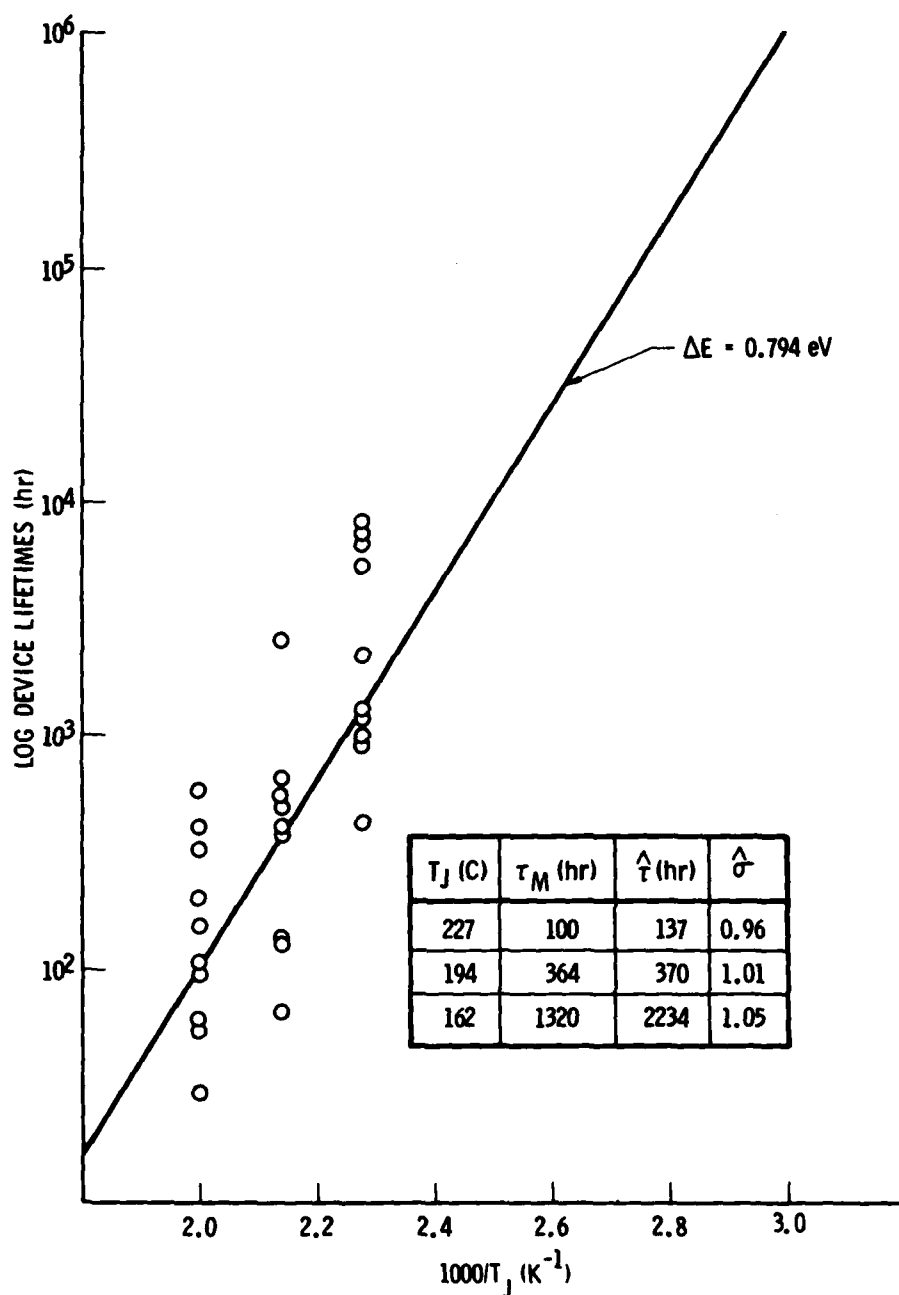
Fig. 1. Arrhenius Presentation of Accelerated Temperature Aging. This presentation was constructed by picking 10 random lifetimes at three different temperatures from a lognormal distribution whose parameters are $\mu = \ln [10^{-6} \exp (0.794/kT)]$ and $\sigma = 1.00$. The exact median lifetimes at the various temperatures are listed, as well as the sample mean lifetime and the logarithmic standard deviation. The solid line represents the Arrhenius dependence of the exact median lifetime, which extrapolates to $\tau_M = 10^6$ hr at an operating temperature of 60°C.

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three different temperatures. The logarithms (to the base 10) of the experimentally measured failure time are plotted as a function of the reciprocal of the absolute "junction" temperature. The scatter in the failure times at constant junction temperature is the result of the random distribution of failure times as expressed by their failure pdf. The average increase in the failure time with increasing reciprocal absolute junction temperature is because their median lifetime follows an Arrhenius dependence.

If one could take a given device to failure at an accelerated temperature, restore the device to its original condition, and then repeat the failure at several different temperatures, the log of the observed failure times plotted as a function of the reciprocal junction temperature would be expected to follow a straight line characterized by a specific τ_o and ΔE. Repeating this experiment with other devices from the same uniform pdf would result in a series of parallel lines, indicating constant failure activation energy. The vertical shift between these Arrhenius dependences is caused by the scatter in the pre-exponential factor, τ_o. Since the pre-exponential factor in an Arrhenius dependence is relatively temperature insensitive, the associated logarithmic variance is also temperature insensitive.

Thermally accelerated aging results are typically analyzed as follows: The failure times observed at a given temperature are plotted on probability paper to determine whether the sample can reasonably be assumed to have come from a distribution whose pdf is lognormal. One's confidence in being able to make such a judgment, of course, increases with the sample size failed at any given temperature. If the failures at each temperature follow a lognormal pdf, the logarithmic variance at each accelerated temperature is estimated to determine whether it is temperature independent. As before, this decision is greatly aided by a large number of failures at each temperature. The logarithmic variance usually ranges from 0.5 to 2.0.[*] A variance of less than

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[*]Reasonable care must be exercised in this type of analysis because of rapid interchange that occurs between naperian base (natural) logarithm and logarithms to the base 10. The natural logarithm is used for σ in this work.

1.0 is characteristic of a reasonably mature and well-controlled process, whereas a variance of greater than 2.0 is indicative of an immature process and is normally rejected out of hand.[1] Having established a reasonably low variance that is supposedly temperature insensitive, we plot the results as indicated in Fig. 1 to decide whether the failures can be fitted with a single activation energy. One's confidence in making such a judgment once again increases with sample size. If it is decided that a single activation energy is justified, constants τ_o and ΔE are found that best describe the data. With these constants, and the logarithmic variance, one can then determine the most appropriate pdf for any desired operating temperature.

This common procedure has a serious flaw that should be emphasized. The failure pdf is estimated as

$$f(\ln t) = \frac{1}{\hat{\sigma}(2\pi)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - \bar{\mu}_N}{\hat{\sigma}}\right)^2\right] \tag{24}$$

where $\bar{\mu}_N$ and $\hat{\sigma}^2$ are the experimenter's best estimate of the extrapolated median log lifetime and the variance. As such, it estimates the "median" cff but not the "average" cff, which can be very different, especially for times much shorter than the extrapolated median lifetime. The important distinction between "median" and "average" estimated cff is the primary motivation of this analysis and is discussed in great detail later.

Our analysis follows the standard treatment given in many books on the theory of statistics. The authors' preference is the early edition of Mood [3]. An excellent bibliography on the theory of statistics is presented in Ref. [4]. An extensive discussion of the lognormal distribution has been provided by Aitchison and Brown [5].

Jordan [6] presented a comprehensive review of the lognormal failure distribution to the analysis of semiconductor component failure. His analysis differed from the present work in that he confined his attention to estimating the statistical error associated with the median lifetime and the logarithmic

variance from a sample of devices all aged to failure at a constant stress temperature. Our primary intention is to estimate the error associated with extrapolating thermally accelerated failures to a lower operating temperature.

An extensive body of literature on the statistical analysis of accelerated stress testing has been contributed by the General Electric (GE) Research and Development Center staff. This work does not appear to be widely discussed within the semiconductor device reliability community as is evident in that it is not referenced in [1], [2], and [6]. Perhaps the explanation for this unfortunate oversight is that the GE group did not employ accelerated temperature aging of semiconductor device results as a vehicle to illustrate the application of their statistical analysis. Regardless of what test results were used, their assumptions (i.e., lognormal pdf/Arrhenius-dependent median log lifetime) are the same as commonly employed in semiconductor device reliability discussions, and their work is, therefore, directly applicable.

The GE analysis is conveniently separated into two parts; complete and censored data. In Nelson's early work [7], the analysis of accelerated temperature aging to complete failure was considered in greater detail than given in Ref. [8]. His main interest was to estimate the confidence limits at the extrapolated median lifetime and the associated standard deviation. However, our main interest is to estimate the probability that a device will fail at a time much less than the extrapolated median lifetime, a subject given only cursory consideration by Nelson. This difference in emphasis was motivated by the final application of the results. We are interested in highly complex systems where the failure rates at times much less than the median lifetime are important, whereas Nelson's interest appears to have been in systems with few components, and, therefore, failure rates near the median lifetime are emphasized. Regardless of this difference in emphasis between the present work and the previous accelerated-aging analysis, there are no basic differences or conflict because they both follow the standard linear regression analysis.

In more recent work from the GE group, the analysis of censored data is considered [9]. An example of censored data in connection with accelerated-

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temperature-stress testing would be where only the first five failure times at
each temperature would be available in the experiment discussed in connection
with Fig. 1. The analysis of censored data is much more complex than the
problem of complete failure data. The subject of censored data is usually
described in more advanced probability and statistical books, such as Kendall
and Stuart's second volume [10]. However, the important subject of the
analysis of censored, accelerated-aging data is outside our present objective.

The correctness of an accelerated-temperature-aging program depends on
systematic and statistical errors. Systematic errors can seriously compromise
the effectiveness of an accelerated stress program. A lower activation energy
failure mechanism invalidates extrapolating the higher temperature failures to
lower temperatures. Although the authors recognized that a thermally
accelerated stress program is very prone to systematic errors, they have
confined this analysis to statistical errors. In this context, the results of
this analysis will provide necessary but, of course, not sufficient criteria
to help determine whether devices from a particular lot, a sample of which was
subjected to thermally accelerated aging, should be installed in a particular
system. The conclusions of this work are valid only under the following
assumptions:

1. The failure pdf is described by single lognormal distribution.

2. The lognormal variance is independent of temperature.

3. The median lifetime follows an Arrhenius dependence whose pre-
    exponential factor and activation energy are independent of
    temperature.

It is recognized that these assumptions apply only to an unbiased, i.e.,
electrically, as opposed to statistically, accelerated stress program and that
the lifetimes should be determined from a biased accelerated-temperature
stress program. The above assumptions are too restrictive in the sense that
if the bias dependence of σ, τ_o, and ΔE are kept constant during the
accelerated stressing and equal to the values at the operational level, the
results of this analysis would also apply to a biased accelerated stress
program. The authors have chosen not to include the biased accelerated stress

assumption not because they feel it is invalid, but because they wish to emphasize the importance of carefully examining the implication of electrical bias during accelerated stress testing. Also, the assumption of a single lognormal pdf rules out the possibility of the existence of a small percentage of early failures.[1]

IV. STATISTICAL ANALYSIS OF ACCELERATED TEMPERATURE AGING

In a single-temperature test, n devices are operated until they all fail. Based on the n different failure times experimentally observed, one desires to estimate the population distribution from this sample distibution. Establishing the validity of an assumed functional form for the failure pdf is outside the scope of this analysis and we proceed directly to estimating the lognormal parameters from the experimentally observed failure times, τ_1, τ_2, τ_3... This will be done by the commonly employed maximum likelihood method. The results of this analysis of a single-failure temperature closely follows the analysis of Jordan[4] and is included here both for completeness and for its usefulness in introducing the more complex analysis to follow.

The experimental observed failure times are converted to the corresponding logarithmic X_1, X_2, X_3..., where each of these samples is from a distribution whose median value and variance we wish to estimate. In the maximum-likelihood method, one designates the deviation of the sample's values from the median value and forms the summation

$$\mathscr{L} = \sum f(X_i - \mu) \tag{25}$$

or

$$\ln \mathscr{L} = - n\ln \left[\sigma(2\pi)^{1/2}\right] - \frac{1}{2\sigma^2} \sum (X_i - \mu)^2 \tag{26}$$

where the summation is performed over the complete sample distribution and is referred to as the log-likelihood function. This function contains all the information available, i.e., the assumed pdf functional form and the experimental results, and, therefore, represents a reasonable starting point in any statistical analysis. In the method of maximum likelihood, it is assumed that reasonable estimators for μ and σ are the values $\bar{\mu}$ and $\bar{\sigma}$, which maximize the maximum-likelihood function or, more conveniently, the log-likelihood function.

Equating to zero the derivatives of the log-likelihood function with respect to both μ and σ provides the desired point estimators, which are

$$\overline{\mu} = \frac{1}{n} \sum X_i \qquad (27)$$

and

$$\overline{\sigma}^2 = \frac{1}{n} \sum (X_i - \overline{\mu})^2 \qquad (28)$$

The estimator for the median value is unbiased; $\overline{\sigma}^2$ is negatively biased but consistent, i.e., consistent in that in the limit of large sample size $\overline{\sigma} \rightarrow \sigma$. The best estimator of σ is, of course,

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum (X_i - \overline{\mu})^2 \qquad (29)$$

If we know σ exactly, we can use Eq. (27) and the self-reproductive property of normal populations to estimate the distribution of the median value as

$$f(\mu') = \frac{1}{\sigma_M (2\pi)^{1/2}} \exp\left[-\frac{1}{2} \left(\frac{\mu' - \overline{\mu}}{\sigma_M} \right)^2 \right] \qquad (30)$$

where

$$\sigma_M^2 = \frac{\sigma^2}{n} \qquad (31)$$

and is commonly referred to as the standard deviation of the mean. This equation expresses the simple truth that the error in the estimated median value follows a normal distribution whose variance decreases linearly with sample size. If a large number of workers were to repeat this experiment, their composite results for $\overline{\mu}$ would follow the above expression except that their median $\overline{\mu}$ would be nearly equal to the exact μ because of the large total sample size involved. The variance in the reported values of $\overline{\mu}$ from all these workers would be nearly equal to the variance expressed in Eq. (31).

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The confidence in one's ability to predict σ from the sample distribution can be estimated by employing the well-known chi-square pdf. For a normal distribution, the random variable

$$\chi^2 = \frac{\sum (X_i - \overline{\mu})^2}{\sigma^2} \qquad (32)$$

has the chi-square distribution with n-1 degrees of freedom.[*] The probability that *a randomly chosen value of* χ^2 *is within some interval (a,b) is*

$$P\left\{ a < \frac{1}{\sigma^2} \sum (X_i - \overline{\mu})^2 < b \right\} = \int_a^b f(\chi^2_{n-1}) d\chi^2_{n-1} \qquad (33)$$

or

$$P\left(\frac{n\overline{\sigma}^2}{b} < \sigma^2 < \frac{n\overline{\sigma}}{a} \right)^2 = \delta \qquad (34)$$

where δ can be found from cumulative chi-square distribution tables once the desired confidence interval is specified.

Knowing neither μ nor σ, one can determine the expected uncertainty in $\overline{\mu}$ by employing the Student-t distribution. Most standard texts on statistical analysis prove that if x is a normally distributed random variable with mean μ and variance σ^2, if χ^2 has a chi-square pdf with k degrees of freedom, and if x and χ^2 are independently distributed, the function

$$t_k = \frac{(x - \mu)/\sigma}{\left[\chi^2/k\right]^{1/2}} \qquad (35)$$

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[*]Our reproducing the complex derivation of the chi-square pdf, or of the Student-t pdf to be shortly introduced, serves little purpose in our primary goal. We desire only to point out the existence and usefulness of these special distributions and refer working reliability engineers to standard text on statistics[3,4] for a more comprehensive discussion.

has the Student-t distribution with k degrees of freedom. We have previously shown that $\bar{\mu}$, i.e., Eq. (30), is the expectation value of μ and, therefore the random variable $\mu - \bar{\mu}$ has a zero mean and variance σ_M^2. Also, since $n\bar{\sigma}^2/\sigma^2$ has been indicated to have a χ^2 distribution of $(n - 1)$ degrees of freedom and is independently distributed from $\mu - \bar{\mu}$,[3]

$$t_k = \frac{(\mu - \bar{\mu})/\sigma_M}{[n\bar{\sigma}^2/(n - 1)\sigma^2]^{1/2}} \tag{36}$$

has the Student-t distribution with $(n - 1)$ degrees of freedom. We can write the confidence level associated with the median value as

$$P\left\{-t(\delta)_{n-1} > t_k = \frac{(\mu - \bar{\mu})/\sigma_M}{[n\bar{\sigma}^2/(n - 1)\sigma^2]^{1/2}}\right\} = \delta \tag{37}$$

or rearranging terms

$$P\left\{\mu < \bar{\mu} - t(\delta)_{n-1}\left[\frac{1}{n(n - 1)} \sum (X_i - \bar{\mu})^2\right]^{1/2}\right\} = \delta \tag{38}$$

where the confidence coefficient δ can be determined from tables of the cumulative Student-t distribution.

Returning once again to the situation where we know σ^2 exactly but not μ; our best estimate as to the failure pdf of the $n + 1$ device taken from the uniform distribution under discussion is

$$f(x) = \frac{1}{\sigma_u (2\pi)^{1/2}} \exp\left\{-\frac{1}{2}\left(\frac{x - \bar{\mu}}{\sigma_u}\right)^2\right\} \tag{39}$$

where

$$\sigma_u^2 = \sigma^2\left[1 + \frac{1}{n}\right] \tag{40}$$

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represents the summation of the intrinsic random variation σ^2 and σ_M^2.
Employing the Student-t distribution, we can estimate the failure pdf of the
n + 1 device by considering

$$t_{n-1} = \frac{(x - \bar{\mu})/\sigma_u}{[n\bar{\sigma}^2/(n - 1)\sigma^2]^{1/2}} \qquad (41)$$

Since the random variable $x - \bar{\mu}$ is normally distributed with zero mean and
variance σ_u^2 and $n\bar{\sigma}^2/\sigma^2$ has a chi-square distribution with n - 1 degrees of
freedom, this expression has the Student-t dependence. We can estimate the
probability of the n + 1 device having a lifetime shorter than some specified
time by

$$P\{-t(\delta)_{n-1} > t = \frac{(x - \bar{\mu})/\sigma_u}{[n\bar{\sigma}^2/(n - 1)\sigma^2]^{1/2}}\} = \delta \qquad (42)$$

which on being rearranged gives

$$P\{\ln\tau_{n+1} < \bar{\mu} - t(\delta)_{n-1}\left(1 + \frac{1}{n}\right)^{1/2}\hat{\sigma}\} = \delta \qquad (43)$$

The statistics of thermally accelerated aging results will now be
analyzed. For this analysis, it is convenient to rewrite the failure pdf as

$$f(x) = \frac{1}{\sigma(2\pi)^{1/2}} \exp\{-\frac{1}{2}\left(\frac{x - \ln\tau_o - \Delta EZ}{\sigma}\right)^2\} \qquad (44)$$

where the Arrhenius relation has been reformulated as

$$\mu = \ln\tau_o + \Delta EZ \qquad (45)$$

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The median log lifetime is a linear function of the variable $(kT)^{-1}$, and the answer to our immediate problem is provided by the well-developed simple linear normal regression technique. Having provided the formalism in the preceding discussion, we may rapidly present the conclusions of this analysis.

From Eq. (44), the log-likelihood function is

$$\ln \mathscr{L} = -n\left[\ln\sigma(2\pi)^{1/2}\right] - \frac{1}{2\sigma^2} \sum\left[X_i - \ln\tau_o - \Delta E Z_i\right]^2 \tag{46}$$

where the couple (X_i, Z_i) represents the experimental log lifetime X_i, observed when the device is operated at Z_i. The summation is, of course, over all devices that are introduced into the accelerated test program. In keeping with our primary objective, we are interested in estimating the lifetime of the $n + 1$ device operated at a normal temperature T_N from the results of an accelerated stress program. Determining the value of σ, $\ln\tau_o$, and ΔE, which maximizes the log likelihood function, yields the estimators

$$\overline{\Delta E} = \frac{\sum(X_i - \overline{X})(Z_i - \overline{Z})}{\sum(Z_i - \overline{Z})^2} \tag{47}$$

$$\overline{\ln\tau_o} = \overline{X} - \overline{\Delta E}\,\overline{Z} \tag{48}$$

$$\overline{\sigma}^2 = \frac{1}{n} \sum(X_i - \overline{\ln\tau_o} - \overline{\Delta E}Z_i)^2 \tag{49}$$

where

$$\overline{X} = \frac{1}{n} \sum X_i \tag{50}$$

and

$$\overline{Z} = \frac{1}{n} \sum Z_i \tag{51}$$

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Summations (47) through (51) are done over all experimentally observed n failures.[*]

Assuming we know σ exactly before the start of this experiment, the estimated distribution of median log lifetime is

$$f(\mu_N') = \frac{1}{\sigma_M(2\pi)^{1/2}} \exp\left\{-\frac{1}{2\sigma_M^2}(\mu_N' - \overline{\ln\tau_o} - \overline{\Delta E}Z_N)^2\right\} \qquad (52)$$

where

$$\sigma_M^2 = \sigma^2 \left[\frac{z_N^2 - 2z_N\overline{z} + \left(\frac{1}{n}\sum z_i\right)^2}{\sum(z_i - \overline{z})^2}\right] \qquad (53)$$

The estimated pdf of the $n + 1$ device at Z_N is

$$f(X_N) = \frac{1}{\sigma_u(2\pi)^{1/2}} \exp\left[-\frac{1}{2\sigma_\mu^2}(X_N - \overline{\ln\tau_o} - \overline{\Delta E}Z_N)^2\right] \qquad (54)$$

where

$$\sigma_u^2 = \sigma^2 + \sigma_M^2 \qquad (55)$$

The above variances can be much larger than the corresponding quantity given in Eqs. (31) and (40) when Z_N is far outside the accelerated temperature range.

An accelerated temperature-stress experiment is not performed at random temperatures, but usually at several specially selected temperatures. Operating at several elevated temperatures is not only less costly, but it

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[*]For a normal pdf, the least-squares method yields identical results.

offers the important advantage of allowing one to evaluate the temperature-independence of the variance. We will now consider a few special sample distributions. Perhaps, the simplest sample distribution is a two-temperature accelerated test with devices equally distributed between the two temperatures. Designating the high- and low-stress conditions as Z_H and Z_L, we rewrite Eqs. (47), (48), (49), (53), and (55) as

$$\overline{\Delta E} = \frac{\overline{X}_L - \overline{X}_H}{Z_L - Z_H} \tag{56}$$

$$\overline{\ln \tau_o} = \frac{1}{2}(\overline{X}_L - \overline{X}_H - \overline{\Delta E}Z_L + \overline{\Delta E}Z_H) \tag{57}$$

$$\overline{\sigma}^2 = \frac{2}{n} \left[\sum_L (X_i - \overline{X}_L)^2 + \sum_H (X_i - \overline{X}_H)^2 \right] \tag{58}$$

$$\sigma_M^2 = \frac{\sigma^2}{n}(L^2 + 1) \tag{59}$$

$$\sigma_u^2 = \sigma^2 \left[1 + \frac{1}{n}(L^2 + 1) \right] \tag{60}$$

where X_L and X_H are the average log lifetimes at the low and high temperatures. In these equations, L represents an effective experimental lever given by

$$L = \frac{Z_N - \frac{1}{2}(Z_H + Z_L)}{Z_L - \frac{1}{2}(Z_H + Z_L)} \tag{61}$$

$$= \frac{Z_N - Z_M}{Z_L - Z_M} \tag{62}$$

and accounts for the increased uncertainity as one moves away from the accelerated temperature range. Note that for L = o the variances, σ_M^2 and σ_u^2 ,are equal to their values in the one-temperature situation. The analogy between this experimental lever arm and a similar relation for the dependence of the accuracy of a rifle on the ratio of firing range to barrel length is obvious.

The question naturally arises if an equal distribution of devices between the two temperatures is statistically the most effective arrangement. Examination of the above variances shows them to be minimized for the sample distribution

$$n_H = \frac{|A|n}{|A| + |B|} \tag{63}$$

and, of course,

$$n_L = n - n_H \tag{64}$$

where

$$A = \frac{1}{2}(1 - L) \tag{65}$$

$$B = \frac{1}{2}(1 + L) \tag{66}$$

The minimized variances are

$$\sigma_M^2 = \frac{\sigma^2}{4n}(|1 - L| + |1 + L|)^2 \tag{67}$$

$$= \frac{\sigma^2}{n} L^2 \qquad (L > 1) \tag{68}$$

and

$$\sigma_u^2 = \sigma^2\left[1 + \frac{1}{n} L^2\right] \qquad (L > 1) \tag{69}$$

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From a practical engineering viewpoint, this minimization is of little value since the effective lever arm is typically L ≈ 6.

Another typical sample distribution is a three-temperature test in which an equal number of components, i.e., $n/3$, are failed at Z_H, Z_M, and Z_L. Our estimated parameters are now

$$\overline{\Delta E} = \frac{\overline{X}_L - \overline{X}_H}{Z_L - Z_H} \tag{70}$$

$$\overline{\ln \tau_o} = \frac{1}{3}(\overline{X}_L + \overline{X}_M + \overline{X}_H) - \overline{\Delta E} Z_M \tag{71}$$

$$\sigma_M^2 = \frac{\sigma^2}{n}(\frac{3}{2} L^2 + 1) \tag{72}$$

$$\sigma_u^2 = \sigma^2 \left[1 + \frac{1}{n}(\frac{3}{2} L^2 + 1) \right] \tag{73}$$

The devices failed at the middle temperature cannot be used to estimate the activation energy. Consequently, for typical experimental lever arms, the middle-temperature failures contribute little to the statistical accuracy of the extrapolated median lifetime, but does, of course, contribute to estimating σ as indicated by Eq. (49).

A uniform distribution is of interest since it approximates a situation in which devices are stressed at many, e.g., ten equally space points along the Z axis. The variance in this case is

$$\sigma_M^2 = \frac{\sigma^2}{n} (3L^2 + 1) \tag{74}$$

with a related expression for σ_u^2. Statistically, two-thirds of the devices are effectively being wasted relative to estimating μ. For any symmetric sample distribution about Z_M, the variance can be written as

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$$\sigma_M^2 = \frac{\sigma^2}{n}(DL^2 + 1) \tag{75}$$

where the D-function is determined by the relative sample distribution throughout the accelerated temperature range. The D-quantity can be thought of as measuring how effectively the samples are distributed. The statistical inefficiency, of course, increases as the devices are located near the center-of-mass of the sample. It is convenient to define the statistical efficiency of an accelerated temperature-stress test as

$$\eta = (DL^2 + 1)^{-1} \tag{76}$$

The statistical efficiency increases as both L and D are reduced.

The chi-square pdf can be employed to estimate the uncertainty in the estimated population variance. This analysis follows the previous one presented for the one-temperature case except that now

$$\chi_{n-2}^2 = \frac{1}{\sigma^2} \sum (X_i - \overline{\ln\tau_o} - \overline{\Delta E} z_i)^2 \tag{77}$$

where the chi-square distribution now has $n - 2$ degrees of freedom. An additional degree of freedom is lost, since at least two experimental points at different temperatures are needed to estimate $\overline{\ln\tau_o}$ and $\overline{\Delta E}$. The confidence level is written

$$P\left\{\frac{n\overline{\sigma}^2}{b} < \sigma^2 < \frac{n\overline{\sigma}^2}{a}\right\} = \int_a^b f(\chi_{n-2}^2) d\chi_{n-2}^2 \tag{78}$$

In evaluating the temperature-sensitivity of σ^2, one should employ not this equation, but Eq. (33), i.e., the confidence level at each of the various accelerated temperatures.

We will now repeat the procedure previously discussed and apply the Student-t distribution to determine the error associated with $\overline{\mu}_N$ and τ_{n+1}

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from accelerated stress results with no _a priori_ knowledge of μ or σ. The quantity

$$t_{n-2} = \frac{(\mu_N - \bar{\mu}_N)/\sigma_M}{[n\bar{\sigma}^2/(n-2)\sigma^2]^{1/2}} \qquad (79)$$

has a Student-t distribution, since $\mu_N - \bar{\mu}_N$ has zero mean and variance σ_M^2, and $n\bar{\sigma}^2/\sigma^2$ is chi-square distributed with $(n-2)$ degrees of freedom.[5] We, therefore, can estimate the probability that μ_N will be lower than any particular value as

$$P\left\{\mu_N < \bar{\mu}_N - t(\delta)_{n-2}\left(\frac{n}{n-2}\right)^{1/2}\frac{\bar{\sigma}\sigma_M}{\sigma}\right\} = \delta \qquad (80)$$

where $\bar{\sigma}$ and σ_M are defined in Eqs. (49) and (53). Since σ_M is proportional to σ, the confidence limits for μ_N can be determined. Transforming this expression into experimental measured parameters, we have

$$P\left\{\mu_N < \overline{\ln\tau_o} + \overline{\Delta E Z}_N - t(\delta)_{n-2}(\hat{\sigma}/nn)^{1/2}\right\} = \delta \qquad (81)$$

where

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum(X_1 - \overline{\ln\tau_o} - \overline{\Delta E Z}_1)^2 \qquad (82)$$

is our unbiased estimator of σ from accelerated temperature aging.

Our primary interest is, of course, in the probability that a randomly selected device operating at normal temperatures will have a lifetime lower than a preselected value. This probability can be found by considering the relation

$$t = \frac{[\ln\tau_{n+1} - \overline{\ln\tau_0} - \overline{\Delta EZ_N}]/\sigma_u}{[n\overline{\sigma}^2/(n-2)\sigma]^{1/2}} \tag{83}$$

which can be shown to have a Student-t distribution employing the same logic as invoked several times before. We write the desired probability as

$$P\left\{\ln\tau_{n+1} < \overline{\ln\tau_0} + \overline{\Delta EZ_N} - t(\delta)_{n-2}\hat{\sigma}\left[1 + \frac{1}{\eta n}\right]^{1/2}\right\} = \delta \tag{84}$$

where $\hat{\sigma}$ is defined in Eq. (82). This relation expresses the probability that τ_{n+1} will be lower than any preselected value in terms of known experimental results. It represents the estimated average cff that should be employed in estimating the system loss to be associated with this device and as such represents completion of our formal statistical analysis.

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V. APPLICATION AND SUMMARY

To demonstrate the application of the preceding analysis, we could take thermally accelerated aging results, such as those illustrated in Fig. 1, and doggedly estimate the various probabilities of interest. We have elected not to proceed in this sterile mode. Instead, we propose considering a very artificial, but instructive, situation. This contrived example has been selected not only because it illustrates the employment and usefulness of the preceding analysis, but also because it clearly shows potential pitfalls that one can encounter in applying accelerated aging results.

Consider the problem of selecting a semiconductor component for a complex space system. Assume that the system analyst has determined that the required device should have an average failure rate of less than 10 FITs, i.e., 10 failures/10^9 hr, over the system useful life of 10^5 hr when the device is operated at a junction temperature of 50°C or less. We can select the device from six different lots, all of which have been through an accelerated temperature-stress program. Although the thermal aging conditions for each of these lots were different (these contrived conditions are listed in Table 1), they all yielded the same extrapolated (50°C) median lifetime and logarithmic variance. In particular, let us assume that the extrapolated median lifetimes are 10^7 hr and that the sample distribution logarithmic standard deviations, $\hat{\sigma} = 1.0$.

Based on the preceding analysis, the results of an accelerated test program extrapolated to a given temperature must be characterized by four parameters — not only $\bar{\mu}_N$ and $\hat{\sigma}$, but also the total number of devices in the accelerated test and the statistical efficiency of the accelerated stress program η. We have listed the statistical efficiency of each experiment in Table 1 with the parameter σ_u, which will prove useful in discussing this situation. Both of these parameters are dependent on the device operational junction temperature, and the values listed in Table 2 are for $T_N = 50$°C.

The probabilities that the lifetime of the device during normal operation will be less than various preselected times, τ', has been calculated employing

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Table 1. Contrived Accelerated Temperature-Aging Experiments

		Extrapolated		
Lot	T_H/n_H	T_L/n_L	η	σ_u/σ
1	250/16	150/16	5.27×10^{-2}	1.26
2	250/16	200/16	8.64×10^{-3}	2.15
3	250/8	150/8	5.27×10^{-2}	1.47
4	250/8	200/8	8.64×10^{-3}	2.87
5	250/4	150/4	5.27×10^{-2}	1.84
6	250/4	200/4	8.64×10^{-3}	3.93

The parameters σ_u/σ and n are calculated with application of Eqs. (60) and (76).

Table 2. Analysis of Contrived Thermal Aging Tests

			$\tau'(hr)$		
Lot	$\delta = 0.25$	$\delta = 0.10$	$\delta = 0.01$	$\delta = 0.005$	$\delta = 0.0005$
0	5.10×10^6	2.77×10^6	9.77×10^5	7.61×10^5	3.72×10^5
1	4.23×10^6	1.92×10^6	4.52×10^5	3.13×10^5	1.01×10^5
2	2.30×10^6	5.98×10^5	5.08×10^4	2.71×10^4	3.94×10^3
3	3.62×10^6	1.38×10^6	2.11×10^5	1.25×10^5	2.27×10^4
4	1.37×10^6	2.11×10^5	5.36×10^3	1.95×10^3	6.91×10^1
5	2.67×10^6	7.06×10^5	3.08×10^4	1.09×10^4	1.73×10^2
6	5.95×10^5	3.49×10^5	4.32×10^1	4.71	6.75×10^{-4}

The τ' is the estimated device lifetime at $T_J = 50°C$ at a confidence level $(1 - \delta)$ (or, equivalently, δ failure probabilities at 50% confidence level).

Eq. (84), remembering $\bar{\mu}_N = \overline{\ln \tau_o} + \overline{\Delta E} Z_N$, and are listed in Table 2. Also included in Table 2 (as lot zero) are the estimated failure probabilities for the case where $\bar{\mu}_N = \ln(10^7 \text{ hr})$ and $\hat{\sigma} = 1.00$ are the exact parameters of a lognormal failure pdf. For the reader's convenience, we display the estimated average cff for three accelerated aging tests as well as the cff for lot zero in Fig. 2.

The estimated cff of lot zero can, of course, be found directly from Eq. (16) since we are assuming that we know τ_M and σ exactly. One can calculate the cff equally as well from Eq. (84) by allowing the sample size to become sufficiently large. With a large sample size

$$t(\delta)_{n-2} \rightarrow t(\delta)_\infty$$

and

$$\left(1 + \frac{1}{\eta n}\right) \rightarrow 1$$

Under the condition of a large sample size, the confidence coefficient δ approaches the standard normal cff.

It is readily apparent that if $\mu_M = \ln(10^7 \text{ hr})$ and $\hat{\sigma} = 1.0$ adequately characterized the failure pdf, the failure rate at times less than the specified useful system life of 10^5 hr would be insignificant. Using the sample distributions $\bar{\mu}_N$ and $\hat{\sigma}$ as the exact μ_N and σ is perhaps the most serious pitfall to be associated with applying accelerated temperature-aging results. For times less than $\bar{\mu}_N$, the analysis of the six contrived accelerated-aging programs indicates an average probability of failure much higher than one estimates using $\bar{\mu}_N$ and $\hat{\sigma}$ as exact failure parameters, i.e., the lot zero case.
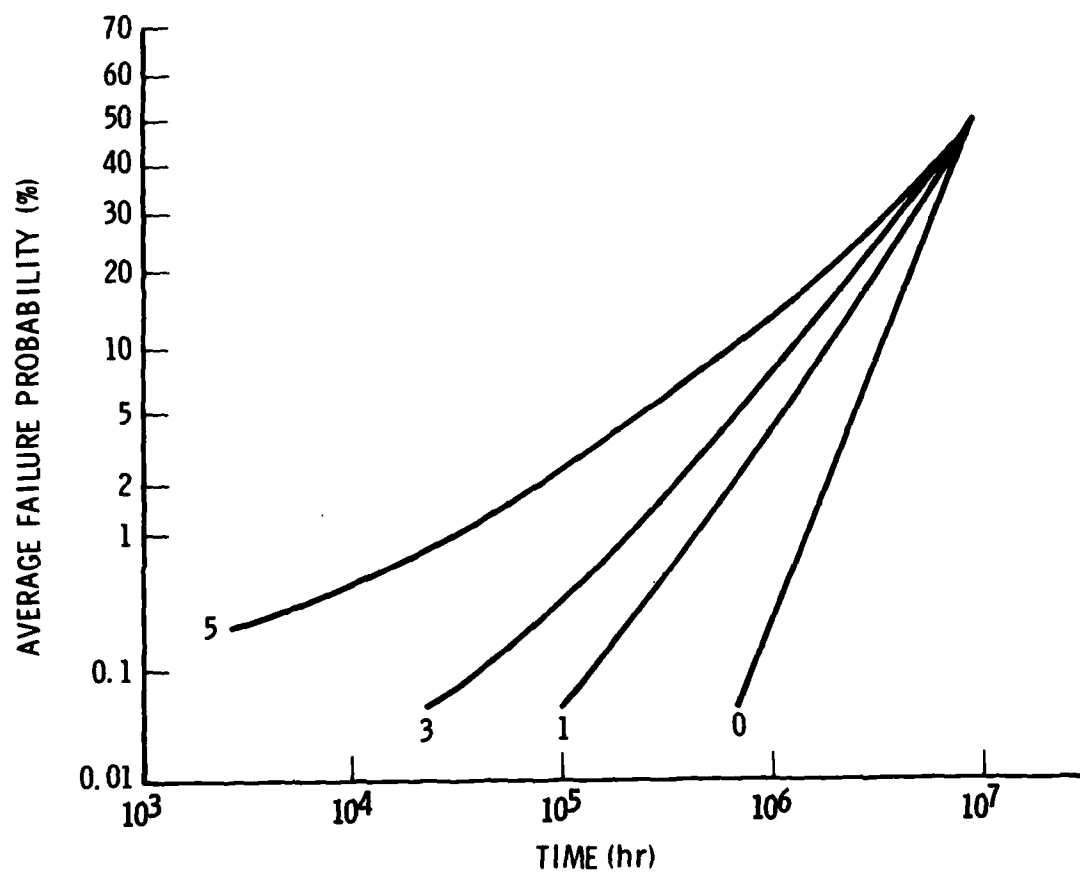
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Fig. 2. Estimated Average Failure Probability as Function of Operating Time at 50°C Inferred From Accelerated Aging. The curve designated 0 is the cff of a lognormal pdf with parameters $\tau_M = 10^7$ hr and $\sigma = 1.00$ (see text).

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The difference between the analysis of lot zero and Eq. (84) is fundamental to our present interest. When one uses $\bar{\mu}_N$ and $\hat{\sigma}$ alone to calculate the cff, one is estimating the median probability of failure at the various operational times and not the average probability of failure as Eq. (84) does. At 10^7 hr, the median and average failure probabilities are equal, but they diverge as one goes to either shorter or longer times. The divergence is a direct result of the uncertainties in the estimates of μ and σ having a greater effect at both short and long times. The probability curves are symmetric about $\bar{\mu}_N$, but since we are not interested in lifetime greater than $\bar{\mu}_N$, only the lower portions are illustrated in Fig. 2.

For lots 1 and 2, the sample sizes are sufficiently large that for $\delta > 0.005$, the error in the estimated cff is caused by the uncertainity in μ and not σ. In other words,

$$t(\delta)_{30} \approx t(\delta)_\infty; \quad \delta > 0.005$$

and the cffs can be approximately represented by a lognormal cff with parameter $\mu_N = \ln(10^7\text{ hr})$ and $\sigma = \hat{\sigma}(1 + 1/\eta n)^{1/2}$. Although both lots 1 and 2 have failed the same number of devices, the uncertainity in the second lot at low values of the cff is much greater because of its longer lever arm or, equivalently, its lower statistical efficiency.

When the sample size is reduced by one-half (lots 3 and 4), the uncertainties in both $\bar{\mu}_N$ and $\hat{\sigma}$ are almost equally important for $0.0005 < \delta < 0.01$. Above $\delta = 0.01$, the uncertainty in μ dominates, whereas below 0.0005, the uncertainty in σ dominates. The exact region between where the uncertainty in μ or σ dominates is, of course, dependent on the relative magnitudes of $t(\delta)_{n-2}$ and σ_u/σ as can be clearly seen by comparing the various lots.

Both the sample size and the statistical efficiency of lots 5 and 6 are so low that the uncertainty in their failure rates at time less than 10^5 hr makes them useless for our assumed requirement. In fact, only lot 1 should be

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considered as meeting the requirement of less than an average of 10 FITs over
a useful system lifetime of 10^5 hr.

As a general rule, it appears more appropriate to assume that the esti-
mated average failure probability follows an exponential failure pdf with
$\lambda = \exp(-\bar{\mu}_N)$ than that $\bar{\mu}_N$ and $\hat{\sigma}$ are the exact value of μ and σ. Extending
this suggestion to many different components, i.e., components from different
accelerated aging experiments, should further increase the accuracy of this
approximation. Considering that it is not difficult to calculate an effective
failure rate with the foregoing analysis when the accelerated aging conditions
are adequately characterized, the assumption of an exponential pdf should be
considered only when the thermal stress results are not properly characterized.

The results of this analysis are best summarized by emphasizing that any
accelerated temperature-aging test should be characterized by more parameters
than just the logarithmic variance of the failed sample distribution and the
extrapolated median lifetime (or, since all users will not operate at the same
junction temperature, $\bar{\tau}_o$ and $\overline{\Delta E}$). This is especially important when one
wishes to estimate the failure rate at times much different than the extrapolated
median lifetime. To properly estimate the average probability of failure at
times much less than $\bar{\mu}_N$, the total number of devices in the accelerated
temperature-aging program should be provided as well as each failure tempera-
ture. These parameters are then used to calculate the probability of failures
at low-level cff, and an estimated average failure rate is calculated. If the
average failure cff meets the system requirements, one is justified in con-
cluding that the statistical requirement has been fulfilled and one should
then turn to the consideration of potential systematic errors in applying
accelerated temperature-aging results -- a subject beyond our limited
objective.

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ABBREVIATIONS AND SYMBOLS

cff   Cumulative failure function

$F(x)$ Cumulative failure function (cff) of random variable (x)

$f(x)$ Failure probability density function (pdf) of random variable (x)

k Boltzmann's constant

L $(Z_N - Z_M)/(Z_L - Z_M)$

ln   Natural logarithm

log   Logarithm to base 10

n Total sample size in accelerated temperature stress (all failed)

n_i Sample size at T_i (all failed)

pdf   Probability density function

T_J Absolute "junction" temperature

T_H Highest accelerated temperature

T_L Lowest accelerated temperature

T_N Normal, i.e., desired, operational temperature

t Time, random variable

$t(\delta)_k$ Student-t distribution with k degrees of freedom associated with confidence coefficient δ

X_i $\ln \tau_i$

x $\ln t$, random variable

Z_i $(kT_i)^{-1}$

Z_M $(Z_H + Z_L)/2$

ΔE Activation energy of Arrhenius-dependent failure mechanism

δ Confidence coefficient

$\lambda(t)$ Device failure rate

μ	Median lifetime/log lifetime
$\bar{\mu}_i$	Mean of sample distribution at T_i
$\bar{\mu}_N$	Mean of sample distribution extrapolated to T_N
σ^2	Variance/log variance
$\bar{\sigma}^2$	Estimator of σ^2
$\hat{\sigma}^2$	Unbiased estimator of σ^2
τ_i	"Experimental" failure time of device
τ_o	Pre-exponential factor of Arrhenius-dependent failure mechanism
X_k^2	Chi-squares distribution with k degrees of freedom

LABORATORY OPERATIONS

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military concepts and systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the Nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

Aerophysics Laboratory: Aerodynamics; fluid dynamics; plasmadynamics; chemical kinetics; engineering mechanics; flight dynamics; heat transfer; high-power gas lasers, continuous and pulsed, IR, visible, UV; laser physics; laser resonator optics; laser effects and countermeasures.

Chemistry and Physics Laboratory: Atmospheric reactions and optical backgrounds; radiative transfer and atmospheric transmission; thermal and state-specific reaction rates in rocket plumes; chemical thermodynamics and propulsion chemistry; laser isotope separation; chemistry and physics of particles; space environmental and contamination effects on spacecraft materials; lubrication; surface chemistry of insulators and conductors; cathode materials; sensor materials and sensor optics; applied laser spectroscopy; atomic frequency standards; pollution and toxic materials monitoring.

Electronics Research Laboratory: Electromagnetic theory and propagation phenomena; microwave and semiconductor devices and integrated circuits; quantum electronics, lasers, and electro-optics; communication sciences, applied electronics, superconducting and electronic device physics; millimeter-wave and far-infrared technology.

Materials Sciences Laboratory: Development of new materials; composite materials; graphite and ceramics; polymeric materials; weapons effects and hardened materials; materials for electronic devices; dimensionally stable materials; chemical and structural analyses; stress corrosion; fatigue of metals.

Space Sciences Laboratory: Atmospheric and ionospheric physics, radiation from the atmosphere, density and composition of the atmosphere, aurorae and airglow; magnetospheric physics, cosmic rays, generation and propagation of plasma waves in the magnetosphere; solar physics, x-ray astronomy; the effects of nuclear explosions, magnetic storms, and solar activity on the earth's atmosphere, ionosphere, and magnetosphere; the effects of optical, electromagnetic, and particulate radiations in space on space systems.

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